The Johns Hopkins Turbulence Databases (*JHTDB*)

FORCED MHD TURBULENCE DATA SET

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The data is from a direct numerical simulation on a 1024^3 periodic grid of the incompressible MHD equations

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \mathbf{j}\times\mathbf{b} + \nu\nabla^2\mathbf{u} + \mathbf{F}$$
$$\partial_t \mathbf{b} = \boldsymbol{\nabla}\times(\mathbf{u}\times\mathbf{b}) + \eta\nabla^2\mathbf{b}$$
$$\boldsymbol{\nabla}\cdot\mathbf{u} = \boldsymbol{\nabla}\cdot\mathbf{b} = 0$$

Here **u** is velocity, $\mathbf{b} = \mathbf{B}/\sqrt{4\pi\rho}$ is the magnetic field in Alfvén velocity units, $\mathbf{j} = \boldsymbol{\nabla}\times\mathbf{b}$ is current, and p is pressure. The external stirring force was a Taylor-Green flow $\mathbf{F} \equiv f_0\,[\sin(k_f x)\cos(k_f y)\cos(k_f z)\,\mathbf{e}_x - \cos(k_f x)\sin(k_f y)\cos(k_f z)\,\mathbf{e}_y]$ applied at modes $k_f = 2$ with an amplitude $f_0 = 0.25$. The magnetic Prandtl number is unity, $\eta = \nu$. The simulation used a pseudo-spectral parallel code in an Elsasser variable formulation[1]. The time integration scheme was slaved 2nd-order Adams-Bashforth, with viscous and resistive terms solved analytically by an integrating factor. The simulation was de-aliased using phase-shift and a $2\sqrt{2}/3$ truncation[2,3]. After the simulation reached a statistically stationary state, 1024 frames of data, which includes the 3 components of the velocity vector, 3 components of the magnetic field vector, and the pressure, were generated and ingested into the database. Also calculated and archived were the 3 components of the magnetic vector potential $\mathbf{a} = \mathrm{curl}^{-1}\,\mathbf{b}$ in the Coulomb gauge $\boldsymbol{\nabla}\cdot\mathbf{a} = 0$. The duration of the stored data is about one large-eddy turnover time.

Simulation parameters:

Domain: $2\pi \times 2\pi \times 2\pi$ (i.e. range of x , y, and z is $[0,2\pi]$)
Grid: 1024^3 , Maximum wavenumber: 482
Viscosity, Resistivity: $\nu = \eta = 1.1 \times 10^{-4}$
Simulation time-step $\Delta t = 2.5 \times 10^{-4}$
Data are stored separated by $\delta t = 2.5 \times 10^{-3}$ (i.e. every 10 DNS time-steps is stored)
Time stored: between t=0 and 2.56 (1024 time samples separated by δt)

Statistical characteristics of turbulence, time averaged over t=0 and 2.56:

	Velocity (w=u)	*Magnetic* (w=b)
Total energy $E_w = \int E_w(k)dk = \frac{1}{2}\langle\lvert\mathbf{w}\rvert^2\rangle$:	$E_u = 7.7\times10^{-2}$	$E_b = 8.5\times10^{-2}$
Dissipation, $\epsilon_w = 2\nu\int k^2 E_w(k)\ dk$:	$\epsilon_u = 1.1\times10^{-2}$	$\epsilon_b = 2.2\times10^{-2}$
rms field component, $w' = \langle\lvert\mathbf{w}\rvert^2/3\rangle^{1/2}$:	$u' = 0.23$	$b' = 0.24$
Taylor microscale $\lambda_w = (15\nu/\epsilon_w)^{1/2}w'$:	$\lambda_u = 8.9\times10^{-2}$	$\lambda_b = 6.6\times10^{-2}$
Taylor-scale Reynolds #, $Re_{\lambda\ w} = w'\lambda_w/\nu$:	$Re_{\lambda u} = 186$	$Re_{\lambda b} = 144$
Kolmogorov time scale $\tau_w = (\nu/\epsilon_w)^{1/2}$:	$\tau_u = 0.1$	$\tau_b = 0.07$
Kolmogorov length scale $\eta_w = (\nu^3/\epsilon_w)^{1/4}$:	$\eta_u = 3.3\times10^{-3}$	$\eta_b = 2.8\times10^{-3}$
Integral scale $L_w = \frac{\pi}{2w'^2}\int k^{-1}E_w(k)\ dk$:	$L_u = 0.56$	$L_b = 0.35$
Large eddy turnover time $T_w = L_w/w'$:	$T_u = 2.43$	$T_b = 1.46$
Cross Helicity $H^C = \int H^C(k)\ dk = \langle\mathbf{u}\cdot\mathbf{b}\rangle$:	$H^C = 1.3\times10^{-3}$	
Magnetic Helicity $H^M = \int H^M(k)\ dk = \langle\mathbf{a}\cdot\mathbf{b}\rangle$:	$H^M = -0.7\times10^{-3}$	

Figures 1, 2, and 3 below show the radial energy spectra, Elsasser spectra, and helicity cospectra, respectively. Figures 4, 5, and 6 show the time series of mean energies, dissipations, and helicities, respectively. Tables with the numerical data plotted in these figures is available in textfiles that can be downloaded from the website.

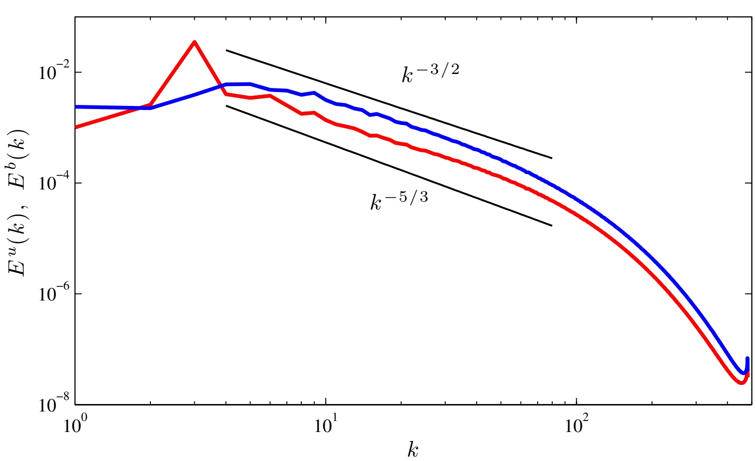

Figure 1: Spectra of velocity (red) and magnetic (blue) fields

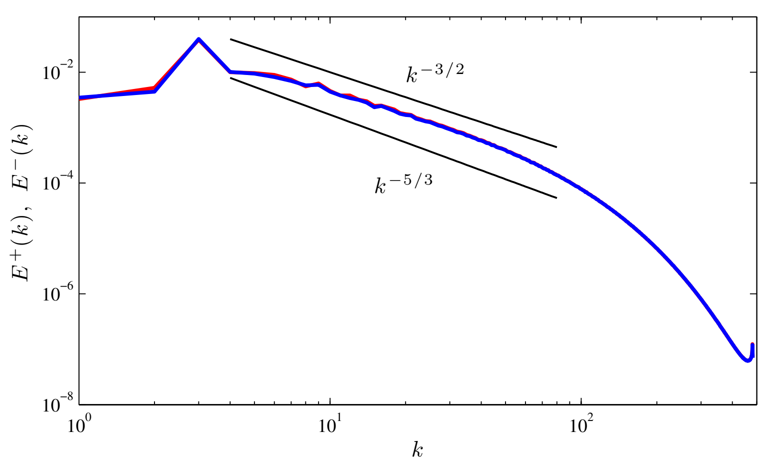

Figure 2: Spectra of Elsasser variables, $\mathbf{z}^+ = \mathbf{u}+\mathbf{b}$ (red) and $\mathbf{z}^- = \mathbf{u}-\mathbf{b}$ (blue)

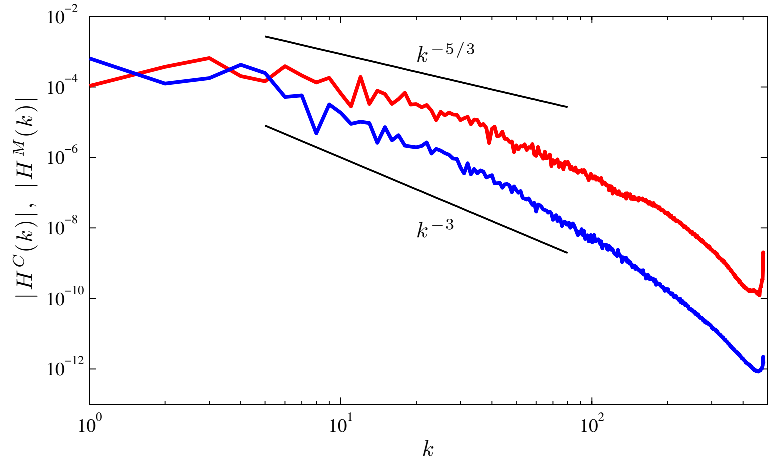

Figure 3: Co-spectra of cross (red) and magnetic (blue) helicity

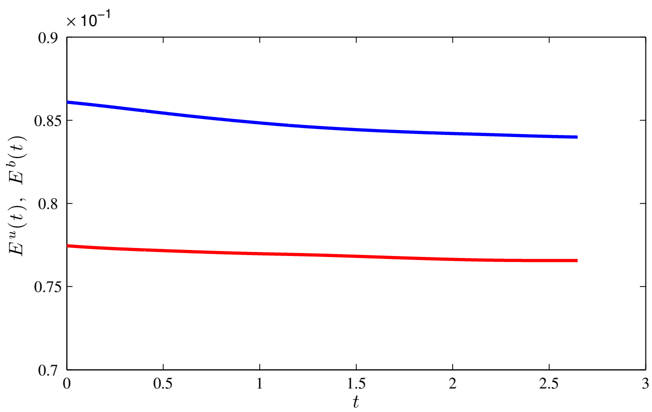

Figure 4: Time-series of kinetic (red) and magnetic (blue) energy

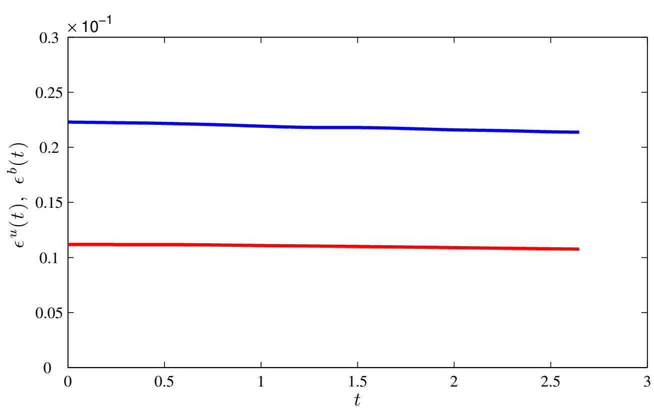

Figure 5: Time-series of kinetic (red) and magnetic (blue) dissipation

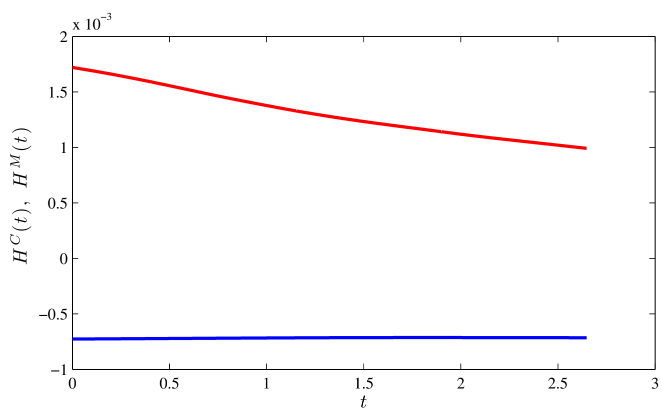

Figure 6: Time-series of cross (red) and magnetic (blue) helicity

References:

1. H. Aluie. Hydrodynamic and Magnetohydrodynamic Turbulence: Invariants, Cascades, and Locality. PhD thesis, The Johns Hopkins University, Baltimore, 2009. http://search.proquest.com/docview/304916341
2. Patterson G.S. and Orszag S.A., "Spectral calculations of isotropic turbulence: efficient removal of aliasing interactions" Phys. Fluids. **14**,2538-2541 (1971).
3. Note: The divergence-free condition in the simulation is enforced based on the spectral representation of the derivatives. The JHTDB analysis tools for gradients are based on finite differencing of various orders. Therefore, when evaluating the divergence using these spatially more localized derivative operators, a non-negligible error in the divergence is obtained, as expected.